

Qualitative indicators of progress and success

Five experienced researchers, Wlodzimierz Kuperberg, Adrian Dumitrescu, Elisabeth Werner, Carsten Schütt, and Shlomo Reisner visited us, each for a period of two months. Unfortunately visits planned for this year by Rom Pinchasi, Pavel Valtr and Tudor Zamfirescu had to be postponed. On the other hand, two senior researchers from the host institute spent altogether nine months at our partner institutes, the Technical University of Catalonia (UPC) in Barcelona, the University College London, Freie Universität Berlin, and ETH Zürich. We also organized two international conferences with over 100 participants. Several joint research projects have been initiated with visitors to Hungary, as well as with members of partner institutions. So far nine papers have been published or accepted for publication, three further papers are submitted for publication. Several research projects are still in the working phase.

Project achievements, scientific highlights

Wlodzimierz Kuperberg finished an earlier started research project with András Bezdek on the problem of occurrence of crossings in thinnest coverings. Their paper is accepted for publication in the journal *Discrete and Computational Geometry*. Wlodzimierz Kuperberg gave a series of talks on this subject. With Gábor Fejes Tóth he continued augmenting the text of the book "Lagerungen in der Ebene, auf der Kugel und im Raum" with comments about the literature after 1972.

Shlomo Reisner worked with Endre Makai Jr., Károly Böröczky Jr. and Mathieu Meyer from our partner institute, the Université de Marne-la-Vallée on the problem of giving lower bounds on the product of the volume of a convex body and its polar body in low dimension. Their work is close to completion in form of a paper. He also worked, maintaining e-mail connection, Mathieu Meyer on two joint projects that they began before. These are connected to shadow movements of convex bodies and log-concave measures.

Adrian Dumitrescu worked on two research projects with János Pach and Géza Tóth. As a partial result towards a conjecture of Fekete and Woeninger they showed that every set of points in the plane with even cardinality at least 8 can be covered by at most two acute vertex disjoint cycles. In another project they investigated the problem of existence of monochromatic empty n -gons in two-colored sets in general position in the plane.

Carsten Schütt and Elisabeth Werner worked with Shlomo Reisner on estimating the product of the volume of a convex body and its polar body in higher dimensions. Exact bounds are known only in special cases, e.g. zonoids, unconditional bodies and polytopes with few vertices. They initiated research of a new special case of the problem. Together with participating members of the host institute they were also investigating new properties of p -affine surface area. They gave talks with the title "Isotropic position and the minimum of the second moment" (Schütt) and "New results on p -affine surface area" (Werner), respectively.

Károly Böröczky Jr. visited for two months the University College London and for three months the Technical University of Catalonia in Barcelona.

During his stay in London in a joint work with Keith Ball (UCL) managed he verified a weak stability version of the Prékopa-Leindler inequality. That inequality is one of the central tools in convexity and in certain areas of probability.

In a joint work with Gergely Ambrus (UCL), they showed that given the area of a convex domain whose centroid is the origin, the area of the so-called centroid domain is minimal for triangles. This statement verifies the planar case of a strong version of the celebrated slicing conjecture.

During his visit at the Technical University of Catalonia in Barcelona Károly Böröczky and Oriol Serra started to study the "sumset" problem in hyperbolic groups. The question is to find good lower bounds on the cardinality of the product with itself of a subset of a hyperbolic group of cardinality n . For Abelian groups, this problem is very well understood, and also for non-commutative groups without torsion. Hyperbolic groups are interesting because they are non-commutative, and their torsion can be controlled. The work is still in progress.

Károly Böröczky and Lluís Vena, a Masters student at UPC in Barcelona, determined the minimal area of a compact hyperbolic surface that can be tiled by embedded regular k -gons of angle 120 degrees for any k larger than six. They also determined the minimal area among orientable compact hyperbolic surfaces. These surfaces are extremal hyperbolic surfaces of minimal area for packings and coverings by equal circular discs of suitable radius.

In addition Károly Böröczky completed various research projects in the stimulating environment at UPC, Barcelona. Together with coauthors, he proved asymptotic formula for the volume approximation in the three space if the number of edges is restricted. Note that similar results were only known if the number of vertices or facets are restricted.

Together with coauthors, he considered the following problem. Given r larger than one, consider the convex body of minimal volume in the three space that contains a unit ball, and whose extreme points are of distance at least r from the center of the unit ball. It was known that the extremal body is the regular octahedron and icosahedron for suitable values of r . They proved that if r is close to one then the typical faces of the extremal body are asymptotically regular triangles.

Gábor Fejes Tóth visited the Freie Universität Berlin and the ETH Zürich, each for two months. He participated there in the famous "Mittagseminars". In Berlin he finished a paper on covering d -dimensional Euclidean space by convex bodies. He also started a new research project on shortest paths among circles. He continued working on this project in Zürich. A preprint on his results is in preparation.

The conference "Intuitive Geometry" held on the week June 30 - July 4, and the Discrete and Convex Geometry Workshop held between July 4 and July 6, in the Rényi Institute offered an excellent opportunity to exchange new ideas. Over one hundred mathematicians from five continents participated in these events. Information about these meetings, in particular the list of participants and abstracts of the talks can be found at the web pages

www.renyi.hu/conferences/intuitivgeometry/

and

www.renyi.hu/conferences/DISCCONVGEO/.

The following talks were held.

At the Intuitive Geometry Conference

Jin Akiyama (Tokai University): Recent Results on Polyhedra

Noga Alon (Tel Aviv University): Weak epsilon-nets: old problems and (some) recent progress

Keith Ball (University College London): A sharp combinatorial version of Vaaler's Theorem

Alexander Barvinok (University of Michigan): The correlation between the row and column sums of a non-negative integer matrix

András Bezdek (University of Auburn, Rényi Institute): On coverings where crossings are unavoidable, and other density related problems

Károly Bezdek (University of Calgary): On some recent progress on ball-polyhedra

Gábor Fejes Tóth (Rényi Institute): Partial Covering of a Convex Domain by Translates of a Centrally Symmetric Convex Disc

Zoltán Füredi (University of Illinois at Urbana-Champaign & Rényi Institute): Covers of closed curves of length two

Peter M. Gruber (Technische Universität Wien): Voronoi type results

Tomas Hales (University of Pittsburg): Some packing problems in two and three dimensions

László Lovász (Eötvös University): Graph representations from matrices

Hiroshi Maehara (Ryukyu University): From line-systems to sphere-systems

Endre Makai (Rényi Institute): Packings of strings of balls and of molecules

Rom Pinchasi (Technion): On some Erdős-type problems in two and three dimensions

Günter Rote (Freie Universität Berlin): Collapse

Francisco Santos (University of Cantabria): Multi-triangulations as complexes of star polygons

Rolf Schneider (Universität Freiburg): Topics in the geometry of random polytopes

Géza Tóth (Rényi Institute): Decomposing multiple coverings

Tudor Zamfirescu (Universität Dortmund): Pushing convex and other bodies through holes

At the Discrete and Convex Geometry Workshop

Gabriela Araujo (UNAM, Mexico): A relation between Cayley Graphs and Polytopes

Ted Bisztriczky (University of Calgary): On convex 4-polytopes

Boris Bukh (IAS, Princeton): Stabbing simplices by points and affine spaces

Robert Connelly (Cornell University): Maximizing the area of unions and intersections of disks

Ludwig Danzer (Universität Dortmund): Some new examples of quasiperiodic tilings

Ferenc Fodor (Szeged University): Random approximation of convex bodies with a reasonably smooth boundary

Jacob Fox (IAS, Princeton): Arrangements of curves and partially ordered sets

Gábor Gévay (Szeged University): Some examples of (n_k) configurations derived from highly symmetric polytopes

Jin-ichi Itoh (Kumamoto University): Thread construction of quadratic (hyper) surfaces

Zsolt Lángi (University of Calgary): On the Hadwiger numbers of centrally symmetric starlike disks

Vitali Milman (Tel Aviv): The abstract concept of duality and some examples

Luis Montejano (UNAM, Mexico): Knesser-Lovász Hypergraphs and transversals to discrete embeddings in affine space

Chie Nara (Tokai University): Space-filling polyhedra with reflectiveness

Márton Naszódi (University of Calgary): Covering a Convex Body by its Homothets of Different Sizes

Deborah Oliveros (UNAM, Mexico): Some realizations and symmetries of the Graphica-hedron

Hellmuth Stachel (TU Wien): Global rigidity of a simplex in Euclidean 4-space with prescribed areas of 2-facets

Ricardo Strausz (UNAM, Mexico): Open problems on separoids

Konrad Swanepoel (TU Chemnitz): Tightest packing and loosest covering in infinite dimensional spaces

Liping Yuan (Beijing): Acute triangulations of surfaces

Asia Ivic Weiss (York University): Map operations and k-orbit maps

Oloff de Wet (University of Pretoria): The Euclidean Steiner Ratio

Teaching and training activities

Five incoming visitors spent a total of 10 months in the Rényi Institute. Each long term visitor gave series of talks presenting new results and methods. They met colleagues and graduate students at the Eötvös University, the Technical University of Budapest and the University of Szeged. They also participated in the two meetings organized in the framework of the DiscConvGeo project. These events offered an excellent opportunity to exchange new ideas.

Dissemination of results

The most important output of the project is publication. So far nine papers written with the support of the DiscConvGeo project have been published or accepted for publication, further three are submitted for publication. Several research projects initiated in the framework of the TOK project are still in progress, some of them close to completion. Part of the work done in the framework of the project was presented at the conference on Intuitive Geometry, at the workshop on Discrete and Computational Geometry, or (and) at other international meetings. Following is a list of papers written with the support of the DiscConvGeo project.

G. Fejes Tóth: Bets partial covering of a convex domain by congruent circles of a given total area, *Discrete and Computational Geometry*, 38 (2007), 312-321.

K. Böröczky, K. Böröczky, Jr., G. Wintsche: Typical faces of extremal polytopes with respect to a thin three-dimensional shell. *Periodica Math. Hung.*, 53 (2006), no. 1-2, 83-102.

K. Böröczky, Jr., Salvador S. Gomis, P. Tick: Volume approximation of smooth convex bodies by three-polytopes of restricted number of edges, *Monatshefte Math.*, 153 (2008), 23-48.

K. Böröczky, K. Böröczky, Jr., C. Schütt, G. Wintsche: Convex bodies of minimal volume, surface area and mean width with respect to thin shells. *Canadian Journal of Mathematics*, 60 (2008), 3-32.

K. Böröczky, Jr., R. Schneider: Stable determination of convex bodies from sections. *Studia Sci. Math. Hung.*, accepted.

K. Böröczky, Jr., R. Schneider: A characterization of the duality mapping for convex bodies. *Geom. Func. Analysis (GAFA)*, accepted.

G. Averkov, E. Makai and H. Martini: Characterizations of centrally symmetry for convex bodies in Minkowski spaces. *Studia Sci. Math. Hung.*, accepted.

G. Fejes Tóth: A Note on Covering by Convex Bodies. *Bull. Canadian Math. Soc.*, accepted.

A. Bezdek and W. Kuperberg: Unavoidable crossings in thinnest plane covering with congruent convex discs, *Discrete and Computational Geomery*, accepted.

K. Böröczky, Jr., R. Schneider: Mean width of circumscribed random polytopes. *CMB*, submitted.

K. Böröczky, Jr., F. Fodor, D. Hug: The mean width of random polytopes circumscribed around a convex body. submitted.

I. Bárány, A. Pór and P. Valtr: Paths with no small angles, submitted to *SIAM J. Discrete Math.*